

SELF-ORGANIZING MAPS WITH HOMOGENEOUS REFRACTORY PERIODS

ANTONIO NEME*, VICTOR MIRELES†

**Grupo de dinámica no lineal y sistemas complejos*
Universidad Autónoma de la Ciudad de México
México, D.F.
México

†*Facultad de Ciencias*
Universidad Nacional Autónoma de México
México, D.F.
México

Emails: antonioneme@gmail.com, syats.vm@gmail.com

Abstract— Self-organizing map (SOM) has been studied as a model of map formation in the brain cortex. However, the original model present several oversimplifications. For example, neurons in the cortex present a refractory period in which they are not able to be activated, restriction that should be included in the SOM if a better model is to be achieved. Although several modifications have been studied in order to include this biological restriction to the SOM, they do not reflect biological plausibility. Here, we present a modification in the SOM that allows neurons to enter a refractory period (SOM-RP) if they are the best matching unit (BMU) or if they belong to its neighborhood. This refractory period is the same for all affected neurons, which contrasts with previous models. By including this biological restriction, SOM dynamics resembles in more detail the behavior shown by the cortex, such as non-radial activity patterns and long distance influence, besides the refractory period. As a side effect, two error measures are lower in maps formed by SOM-RP than in those formed by SOM.

Keywords— Self-organization; refractory period; map formation

1 Introduction

The self-organizing map (SOM) is presented as a model of the self-organization of neural connections, which is translated in the ability of the algorithm to produce organization from disorder [1]. One of the main properties of the SOM is the ability to preserve in the output map those topographical relations present in input data [2]. This property is achieved through a transformation of an incoming signal pattern of arbitrary dimension into a low-dimensional discrete map (usually one or two-dimensional) and to adaptively transform data in a topologically ordered fashion [3, 2]. Each input data is mapped to a single neuron in the lattice, the one with the closest weight vector to the input vector, or best matching unit (BMU). The SOM preserves relationships during training through the learning equation, which establishes the effect each BMU has in any other neuron. Weight neurons are updated accordingly to:

$$w_n(t+1) = w_n(t) + \alpha_n(t)h_n(g,t)(x_i - w_n(t)) \quad (1)$$

Where $\alpha(t)$ is the learning rate at time t and $h_n(g,t)$ is the neighbourhood function from BMU neuron g to neuron n at time t . In general, the neighbourhood function decreases monotonically as a function of the distance from neuron g to neuron n . The SOM tries to preserve relationships of input data by starting with a large neighbourhood and reducing it during the course of training [2, 4].

As pointed out by Ritter [3], SOM and related algorithms share the idea of using a deformable lattice to transform data similarities into spatial relationships. The lattice is deformed by applying learning equation (1).

Although SOM has been widely applied in data visualization and clustering, it has also been studied as a model of the brain cortex. For example, in [5, 6] it has been studied to understand the map formation in visual cortex, and in [2] as a model of brain maps from sensorial areas to cortical regions. However, SOM fails to reproduce the activity patterns present in the cortex [16, 17], although some variants, as the one proposed in [8], in which a modification in the kernel is considered to include surround inhibition, achieve the formation of pinwheel patterns similar to those observed in the visual cortex. In these models, however, influence from BMUs to neighbors is radial and symmetrical. In [19] non-radial patterns of activity from BMU to its neighbors are reported as a consequence of differentiated influence based on the relative frequency each BMU includes neurons as neighbors. In [18], a recursive rule allows non radial neighborhood adaptation as a consequence of the pulling from BMUs to the direct neighbors whereas these neighbors further pull their neighbors.

Concepts analogous to the refractory period have also been included in the SOM. For example, in [12] the BMU is relaxed in order to modify the magnification exponent. In [20] a traveling wave of activity induced in BMU allows its

neighbors to be more likely to be the next BMU, which can be interpreted as a refractory period for those neurons not included in the wavefront. In [21] an activation memory is defined for each neuron, in order to define the new active neuron, and a modification in the BMU selection mechanism is presented, so if the memory parameter is high, the previous winner neuron will win again unless another neuron matches very close the input data, which, again, may be seen as a refractory restriction for those neurons with a low memory parameter. When applied for time sequences analysis, the SOM incorporates a kind of refractory period that allows the identification of temporality [13].

In these modifications, the refractory time depends on the weight modification which may not be a biologically realistic behavior [15]. Here, we study the effects of including a refractory period that does not depend on this or any other quantity in the neurons for the SOM, the SOM-RP, and show that it is possible to obtain maps equivalent to those obtained with the SOM.

2 The self-organizing map with refractory period (SOM-RP)

Once an input vector is mapped to a neuron, the later becomes a BMU and is activated, as well as its neighbors. Biologically, the activation of a given neuron is achieved through the electrical opening of ion channels which, due to concentration gradients and charge differences between the out and inside of the cell, drive positive ions into the cell. This changes the potential (inside relative to outside) of the cell, from its resting potential to the activity one [14].

For this change of potential to take place, a fair amount of positive ions must flow into the cell, thus reversing the concentration gradient of this ions for a moment. Since the diffusion force plays an important role in the activation process, this cannot take place until the concentration difference is reestablished, so when the proper channels open, positive ions will indeed flow into the neuron. The time in which the necessary concentration is not present is referred to as the absolute refractory period, referred here as refractory period [15, 14]. While there also exists a relative refractory period in which the neuron can become active but needs a far greater stimulus to do so, we will not consider it in the present work.

In this work, we are interested in the SOM capabilities to form maps that resemble the input space distribution even when some of the neurons in the network are not able to learn for a given period of time, which affects the SOM dynamics as the weight folding may follow other routes.

We propose a modification that allows active neurons to enter a refractory period. By active neurons we mean BMU and those neurons within

its neighborhood. The studied modifications are two: in the first one, each BMU and a subset of its neighbors enter a refractory period in which they are not able to be affected by any other BMU for a given time. Two variables are defined here: τ and d . τ is the refractory period and is defined as the number of input vectors for which an active neuron is not able to learn through eq. (1). τ does not depend on the weight update value or any other variable and is the same throughout the learning process and for all affected neurons. d is the radius of a hypersphere in the lattice centered at the BMU, which defines the set of neurons that will fall into refractory period. If d is greater than the actual neighborhood width, then d is set to that width.

In the second modification studied in this work, BMUs and all its neighbors present a maximum number of times they can be affected before entering the refractory period, named c . Once this maximum is achieved, they enter the refractory period and stay there for τ vectors. That is, c acts as a delay for the neurons to enter refractory period. Once again, τ is the same for all sleeping neurons.

For both modifications, the refractory period is the same for all neurons, which contrasts with the models mentioned in the previous section.

3 Results

In order to study SOM-RP dynamics, several thousand maps were formed for six data sets: spiral, random and unitary circumference (2-dimensional); iris (4-dimensional); Mexican elections (ME) (6-dimensional) and ionosphere (34-dimensional) data sets.

In order to verify self-organization in the SOM-RP two error measures were quantified and compared to the error measures present in the maps formed by eq (1). Although there are several error measures for the maps obtained by SOM and there is no solid definition of the energy function [9, 10, 11], the topographic error (TE) as well as the error quantization (EQ) were the error measures quantified for the obtained maps, as they are good measures of the quality of topographic mapping and vector quantization. In order to test sensitivity and self-organization, several thousands of experiments were made for two lattice sizes, $N \times N$ ($N = 20$ and 30), as well as for the initial learning parameter $0 < \alpha(0) \leq 1$ and for the initial neighborhood size $1 < h_n(g, 0) \leq N$. Practically, for each

- learning set (circumference, spiral, random, iris, ionosphere, ME data),
- number of epochs (between 1 and 30),
- τ (between 0 and 25)

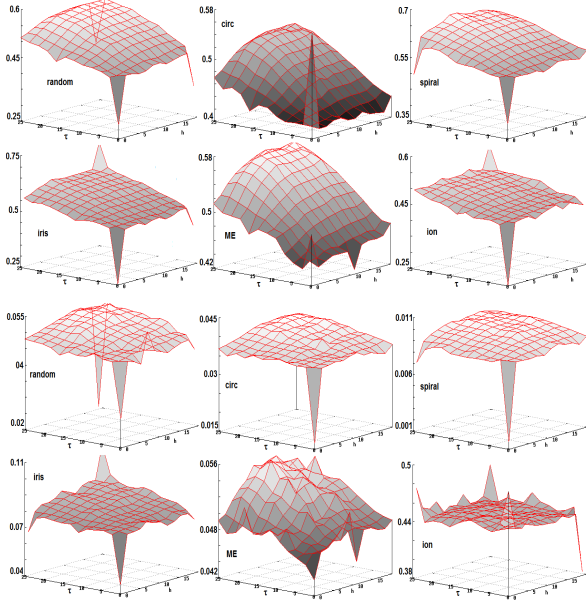


Figure 1: TE (top) and EQ (bottom) for modification 1 for the six data sets.

- d (between 0 and 25) (for modification 1)
- c (between 0 and 25) (for modification 2)

the initial learning parameter $\alpha(0)$ was chosen randomly from $(0, 1]$ as well as the initial neighborhood width was chosen from $[1, N]$. The final learning parameter was 0.0001 ($\alpha(r) = 0.0001$) whereas the final width was decreased to 0 by an exponential function. In both modifications, if $\tau = 0$ then SOM-RP is reduced to SOM.

Fig. 1 shows TE and EQ as a function of τ and d for the first modification, whereas fig. 2 presents TE and QE as a function of c and τ for the second modification. What is observed is that error measures are in general lower for SOM, corner $(0,0)$, than for SOM-RP. However, it is important to notice that this is condensed information for several thousand maps, with different epoch numbers, as well as different neighborhood width and α values.

Although TE and EQ are in general greater in SOM-RP than in SOM, Let us consider, as shown in fig. 3, only the 5% of maps with the lowest TE values, here maps formed by SOM-RP ($\tau > 0$) are much more frequent than those formed by SOM. In some data sets, none of the low-error maps were obtained for several values of both τ and d (or c for modification 2) (see fig. 4).

On a 30×30 lattice we performed thousands of simulations of both SOM and SOM-RP and examined how frequent a given TE was achieved. The results are shown in fig 5.

As we can see, on all cases, the traditional SOM presents spikes in the frequency histogram

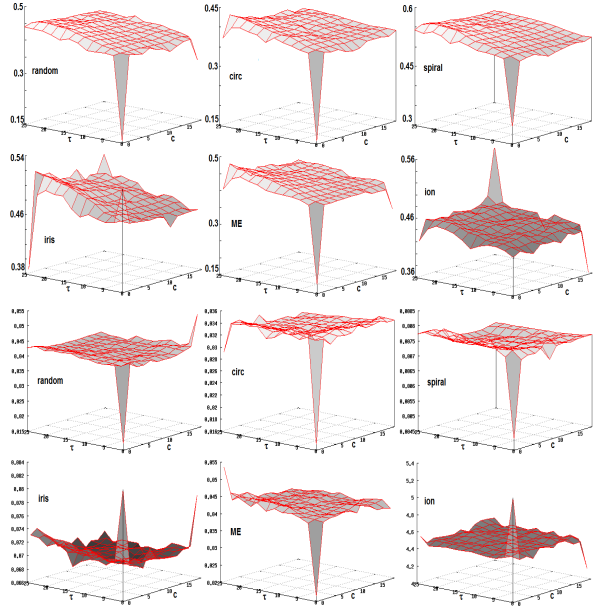


Figure 2: TE (top) and EQ (bottom) for modification 2 for the six data sets.

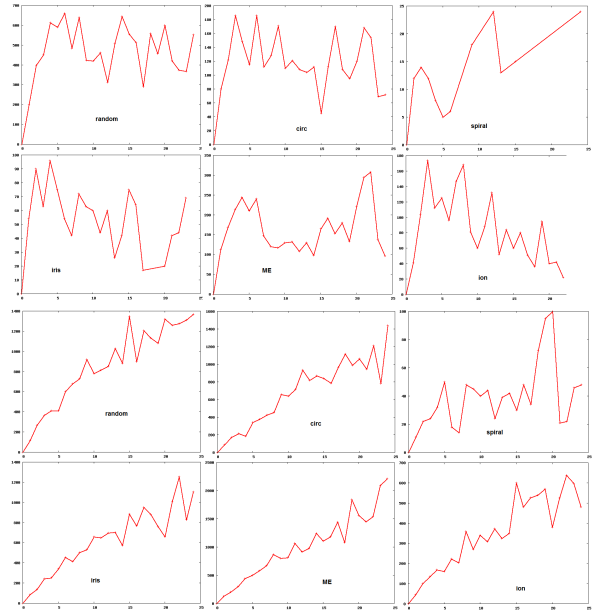


Figure 3: The 5% of the maps with lowest TE are considered. τ is indicated in the x axis ($\tau = 0$ is the original SOM), while in the y axis is considered the number of maps for that τ that are in the group of the maps with very low errors. The first six figures are for modification 1 and the last six for modification 2.

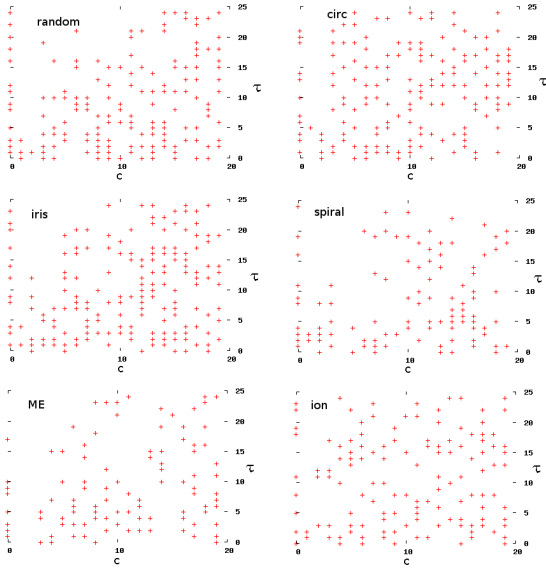


Figure 4: τ and c for the 5% of the maps with the lowest TE for the six data sets and lattice of size 20×20 .

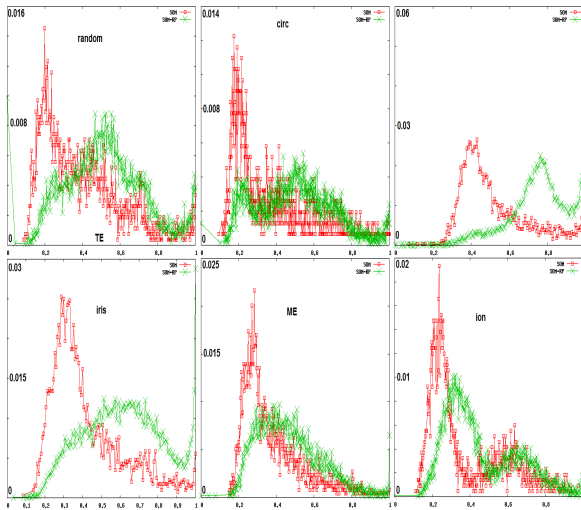


Figure 5: Frequency of maps (y axis) for TE (x axis) for lattices of size 30×30 for SOM (squared) and SOM-RP (cross).

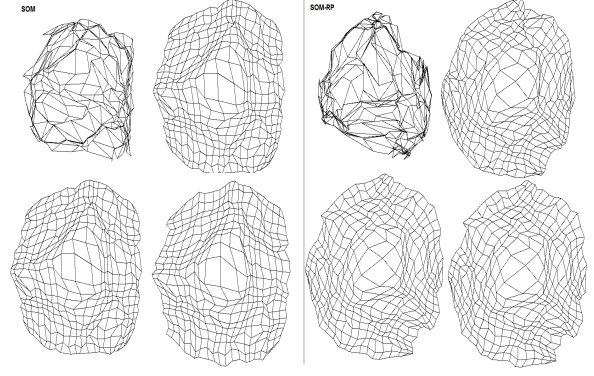


Figure 6: Weight folding in the SOM (left) and in the SOM-RP (right) for $t = 2, 4, 6, 10$ for the ring data set (only included as an example for weight folding). It is observed that the SOM-RP presents, in general, smooth borders that fit the input vectors. SOM-RP parameters were $\tau = 2$ and $d = 2$.

which are further left than the spikes present in the SOM-RP frequency histogram. This means that the most likely TE of the SOM is lower than the most likely TE of the SOM-RP.

This drawback is of importance if a small number of maps are made, however, as can be seen above, if a large enough number is made and the map with lowest TE is chosen, then the SOM-RP is very likely to produce a map with significantly lower TE than SOM.

Folding in the SOM-RP is affected as shown in fig. 6. It is observed that the SOM-RP approximates better the ring data set as it shows smooth borders which contrast with the borders in the SOM weights. This is a consequence of the refractory period in which neurons enter after being affected. When a neuron enters this period, some of its neighbors may be able to learn the new input. Thus, the network learning may be improved if some of the neurons do not take part in the process for some periods of time. It is also observed that the bottom-right corner is not properly folded, once again, as a consequence of the refractory period. The refractory period may help the network to properly fold, but, if τ is large enough and the area of influence is also large, then the folding may be disrupted.

The activity patterns in the SOM are radial and symmetrical, which is different from the patterns formed in SOM-RP, as shown in fig 7. The BMU affects all its neighbors, but only a subset of them (defined by the d parameter, $d = 3$ in the example) will become inactive for τ input vectors ($\tau = 2$). Once τ input stimulus are mapped, the refractory neurons become susceptible and may be affected by BMUs or even become BMU. Neurons in BMU's neighborhood might not modify their weight vector, as they might be in a refractory period. Closer neurons to BMU may be refractory,

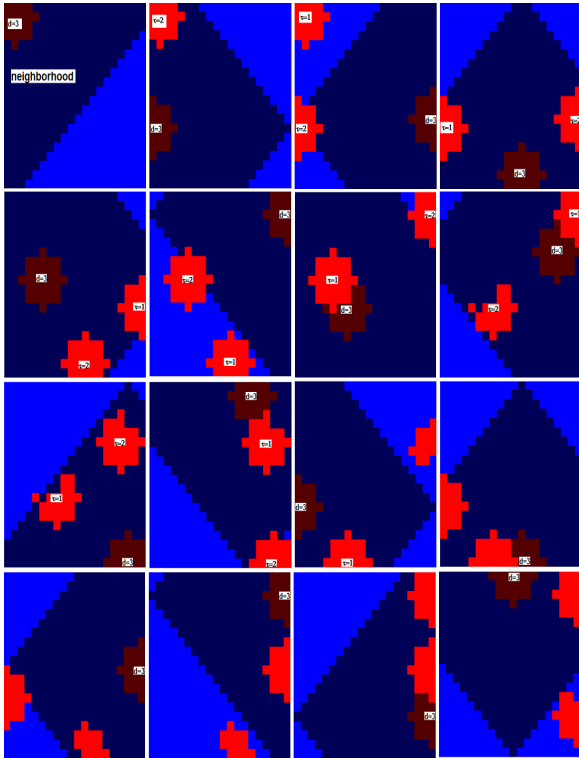


Figure 7: Activity patterns in the SOM-RP for consecutive times. Non radial patterns of activity are formed because of the refractory period. Here, the parameters for the SOM-RP were $\tau = 2$ and $d = 3$ in a 20x20 lattice.

while farther neurons (also included in neighborhood) may be affected. This long-distance effect has biological foundations.

4 Discussion and conclusions

Map formation is possible when homogeneous refractory periods are included in neurons. We are interested in studying the properties of SOM when a refractory period is included in its neurons. Although it is not possible to give a recipe for τ , d and h values, we have shown it is possible that the map folds properly to approximate input space, which is an important restriction to be included in the SOM if it is to be studied as a more realistic model of the brain cortex.

In nature, refractory periods are important as they allow neurons to repolarize and become susceptible for further activation. In general, it had been identified as a restriction in neurons that should not be considered in artificial models, no matter they are supposed to explain the general aspects of self-organization in the brain. Here, we have incorporated a homogeneous refractory period and the results are, we believe, interesting, in the sense that are equivalent (in terms of errors) to those obtained by SOM, but achieved by following a different and more realistic route.

The fact that learning took place in the SOM-

RP, reaffirms the fact that the learning process is a distributed one, for modifications in 'small' regions of the lattice do not affect the overall behaviour of the map. This, of course, can be seen with many other modifications to the original SOM. It is however of interest, that the overall behaviour of the map is little affected by phenomena which are not instantaneous: for several values of τ the properties of the SOM are not dramatically altered, which suggests that the distribution of information and processing capabilities in the SOM are robust enough as to go around lasting obstacles.

The existence of a refractory period drives the BMU around the grid, forcing it to fall into non-recently visited sites. Since in general τ will be significantly smaller than the number of input vectors, this is translated into further spreading the neurons associated with each stimulus. This will in turn make the convergence of the mapping slower. This is not necessarily a drawback, for it also makes the distribution of the weight vectors more uniform in the input space, thus allowing a better mapping.

Non-symmetrical activity patterns are present in the cortex, but the SOM fails to reproduce them. With the proposed SOM-RP, those patterns, as well as long-distance influence, are achieved.

References

- [1] Cottrell Cottrell, M. Fort, J.C., Pagés, G. Theoretical aspects of the SOM algorithm. *Neurocomputing* 21 (1998) 119-138.
- [2] Kohonen Kohonen, T. *Self-Organizing maps*. 3rd. ed. Springer-Verlag. (2000).
- [3] Ritter Ritter, H. *Self-Organizing Maps on non-euclidean Spaces Kohonen Maps*, 97-108, Eds.: E. Oja and S. Kaski, (1999).
- [4] Erwin Erwin, Obermayer, K. Schulten, K. *Self-organizing maps: Ordering, convergence properties and energy functions*. *Biol. Cyb.* 67 (1992) 47-55.
- [5] Mayer Mayer, N., Herrmann, J., Geisel, T. *Retinotopy and spatial phase in topographic maps*. *Neurocomputing* 32-33 (2000) 447-452.
- [6] Bednar Bednar, J., Kelkar, A, Miikkulainen, R. *Scaling self-organizing maps to model large cortical networks*. *Neuroinformatics*. (2004).
- [7] Kohonen Kohonen, T. *Self-organizing neural projections*. *Neural Networks* 19 (2006). 723-733.
- [8] Liljeholm Liljeholm, M., Lin, A., Ozdzyński, P., Beatty, J. *Quantitative analysis of kernel*

- properties in Kohonen's self-organizing map algorithm: Gaussian and difference of Gaussians neighborhoods. *Neurocomputing*. 44-46 (2002) 515-520.
- [9] Goodhill Goodhill, G., Finch, S., Sejnowski, T. Quantifying neighborhood preservation in topographic maps. 3rd. Joint Symp. in neural comp. (1996) 61-82.
- [10] Kiviluoto Kiviluoto, K. Topology preservation in Self-Organizing maps. Proc. ICNN96, IEEE Int. Conf. on Neural Networks. (1996).
- [11] Bauer Bauer, H., Herrmann, M., Villmann, T. Neural maps and topographic vector quantization. *Neural networks* 12 (1999) 659-676.
- [12] Claussen Claussen, J. Winner-relaxing self-organizing maps. *Neural computation* 17 (2006) 996-1009.
- [13] Barreto Barreto, G.Araujo, A. Time in self-organizing maps: an overview of models. *Int. J. of computer research* Vol 10 N. 2 (2001) 235-295.
- [14] Thompson Thompson, R. The brain: a neuroscience primer. Worth pub. (2000).
- [15] Koch Koch, C. Biophysics of computation and information processing in single neurons. Oxford University press (1998).
- [16] Koulakov Koulakov, A., Chklovsky, D. Orientation preference patterns in mammalian visual cortex: a wire length minimization approach. *Neuron* 29 (2001) 519-527.
- [17] Goodhill2 Goodhill, G., Cimponeriu, A. Analysis of the elastic net model applied to the formation of ocular dominance and orientation columns. *Network: comput. neural systems* 11 (2000). 153-168.
- [18] Lee Lee, J., Verleysen, M. Self-organizing maps with recursive neighborhood adaption. *Neural Networks* 15 (2002) 993-1003.
- [19] Neme Neme, A., Miramontes, P. A parameter in the SOM learning rule that incorporates activation frequency. *ICANN* (1) (2006) 455-463.
- [20] Principe Principe, J., Euliano, N., Garani, S. Principles and networks for self-organization in space-time *Neural Networks* 15 (2002) 1069-1083.
- [21] Chappell Chappell, G., Taylor, J. The temporal Kohonen map. *Neural Networks* 6 (1993) 441-445.